

The galaxy-wide IMF - from star clusters to galaxies

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Abstract. Over the past years observations of young and populous star clusters have shown that the stellar initial mass function (IMF) can be conveniently described by a two-part power-law with an exponent $\alpha_2 = 2.3$ for stars more massive than about $0.5 M_\odot$ and an exponent of $\alpha_1 = 1.3$ for less massive stars. A consensus has also emerged that most, if not all, stars form in stellar groups and star clusters, and that the mass function of these can be described as a power-law (the embedded cluster mass function, ECMF) with an exponent $\beta \approx 2$. These two results imply that the integrated galactic IMF (IGIMF) for early-type stars cannot be a Salpeter power-law, but that they must have a steeper exponent. An application to star-burst galaxies shows that the IGIMF can become top-heavy. This has important consequences for the distribution of stellar remnants and for the chemo-dynamical and photometric evolution of galaxies. In this contribution the IGIMF theory is described, and the accompanying contribution by Pflamm-Altenburg, Weidner & Kroupa (this volume) documents the applications of the IGIMF theory to galactic astrophysics.

1. Intro

The stellar initial mass function (IMF) defines the ratio of low-mass stars ($< 1 M_\odot$), which do not contribute to the chemical evolution over a Hubble time but lock-up baryonic matter, to high-mass stars ($> \text{few } M_\odot$), which power the interstellar medium and enrich it with metals through AGB-winds and supernovae. It further determines the mass-to-light ratios of stellar populations and influences the dynamical evolution of star clusters and whole galaxies.

Over the last years it had become clear that star formation takes place mostly in embedded clusterings (Lada & Lada 2003; Allen et al. 2007), each cluster or group containing a dozen to many millions of stars (Kroupa 2005). Within these clusters stars appear to form following the canonical IMF, $\xi(m) \propto m^{-\alpha}$, with a slope of 1.3 for stars with $m \leq 0.5 M_\odot$ and the Salpeter/Massey-slope of 2.35 for $m > 0.5 M_\odot$ stars.

A result of clustered star-formation is that the composite stellar population in a galaxy, which results from many star-forming events, is the sum of the dissolving star clusters. Thus the integrated galactic initial mass function (IGIMF) is the sum of all the IMFs of all the star clusters (Kroupa & Weidner 2003; Weidner & Kroupa 2005).

But (young, embedded) star clusters also follow a mass function. The embedded cluster mass function (ECMF) has been found to be a power-law, $\xi_{\text{ecl}} \propto M_{\text{ecl}}^{-\beta}$, with a

rather constant slope of ≈ 2 for largely different environments from the quiescent solar neighbourhood to the vigorously star-forming Antennae galaxies (Lada & Lada 2003; Hunter et al. 2003; Zhang & Fall 1999).

Additionally, it appears that star clusters limit the mass of the most-massive star that can form within them. Low-mass clusters are unable to form very massive stars. This follows not only from observations but is a necessary logical statement given that a finite mass reservoir is distributed over the stellar population with an invariant IMF shape. This results in a relation between cluster mass and the most-massive star (Weidner & Kroupa 2004, 2006; Weidner et al. 2010a).

A direct consequence is that the IGIMF is steeper than the individual canonical IMFs in the actual clusters, hereby immediately explaining why $\alpha_{3,\text{field}} = 2.7 > \alpha_3 = 2.35$, where $\alpha_{3,\text{field}}$ is the slope of the IMF derived by Scalo (1986) and Reid et al. (2002) from OB star counts in the Milky Way field, and $\alpha_3 = 2.35$ is the Salpeter (1955) index. This is due to the fact that low-mass star clusters are numerous but can not have massive stars.

A mathematical description of how the IGIMF¹ can be calculated is given in § 2, while § 3 will discuss observational consequences of the IGIMF for the stellar populations of galaxies.

2. Mathematical formulation of the IGIMF

In order to describe the IGIMF mathematically, three ingredients are necessary. The IMF within star clusters (described in § 2.1), how the cluster mass limits its most-massive star (§ 2.2) and how the cluster mass function (ECMF) is defined by the star-formation rate (SFR) of a galaxy (§ 2.3). Table 1 gives an overview of the step-by-step development of the IGIMF theory.

2.1. The stellar IMF in star clusters

An arbitrary multi-power law distribution function is

$$\xi(m) = k \begin{cases} k' \left(\frac{m}{m_H}\right)^{-\alpha_0} & , m_{\text{low}} \leq m < m_H, \\ \left(\frac{m}{m_H}\right)^{-\alpha_1} & , m_H \leq m < m_0, \\ \left(\frac{m_0}{m_H}\right)^{-\alpha_1} \left(\frac{m}{m_0}\right)^{-\alpha_2} & , m_0 \leq m < m_1, \\ \left(\frac{m_0}{m_H}\right)^{-\alpha_1} \left(\frac{m_1}{m_0}\right)^{-\alpha_2} \left(\frac{m}{m_1}\right)^{-\alpha_3} & , m_1 \leq m < m_{\text{max}}. \end{cases} \quad (1)$$

The IMF in star clusters has been found to be conveniently described by the following set of indices.

$$\begin{aligned} \alpha_0 &= +0.30 & , & 0.01 \leq m/M_\odot < 0.08, \\ \alpha_1 &= +1.30 & , & 0.08 \leq m/M_\odot < 0.50, \\ \alpha_2 &= +2.30 & , & 0.50 \leq m/M_\odot \leq m_{\text{max}}(M_{\text{ecl}}), \end{aligned} \quad (2)$$

where $dN = \xi(m) dm$ is the number of stars in the mass interval m to $m + dm$ and m_{max} is the mass of the most-massive star in a cluster which is regulated by the cluster mass, M_{ecl} . The exponents α_i represent the standard or canonical IMF (Kroupa

¹A tool to calculate an IGIMF and to fit it with a multi-part power law as required as the input IMF for PEGASE has been presented in Pflamm-Altenburg et al. (2009) and can be downloaded from www.astro.uni-bonn.de

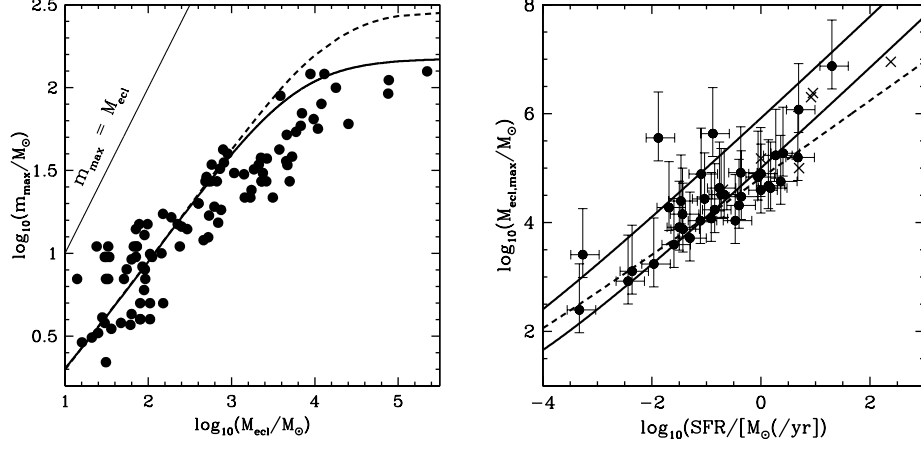


Figure 1. Left: The most-massive star, m_{\max} , vs the embedded cluster mass, M_{ecl} . The dots are observations from Weidner et al. (2010a) and the solid line is a numerical solution to eqs. 4 and 3 assuming $m_{\max*} = 150 M_{\odot}$, while for the dashed line $m_{\max*}$ is $300 M_{\odot}$. Right: Maximum cluster mass versus global star-formation rate (SFR), both in logarithmic units. Filled dots are observations compiled by Weidner et al. (2004) with error estimates and the crosses are additional recent data points. The top solid line is the result of eqs. 11 and 12 with $\beta = 2.0$ and $\delta t = 10$ Myr and the bottom solid line of $\beta = 2.0$ and $\delta t = 1$ Myr, while the dashed line assumes $\beta = 2.4$ and $\delta t = 10$ Myr.

2001, 2002), and k' signifies the discontinuity near the brown dwarf / stellar mass limit (Thies & Kroupa 2007). Note that $\alpha_3 \approx \alpha_2$ so that the canonical IMF is excellently described by a very convenient two-part power law distribution function.

2.2. The most-massive star in a star cluster

In order to analytically determine the mass of the most-massive star in a cluster, the fact that there must be one most-massive star is used,

$$1 = \int_{m_{\max}}^{m_{\max*}} \xi(m) dm, \quad (3)$$

where $m_{\max*}$ is the fundamental upper mass limit for stars which was found to be $m_{\max*} \approx 150 M_{\odot}$ (Weidner & Kroupa 2004; Figer 2005; Koen 2006), though new indications exists that it may be as high as $300 M_{\odot}$ (Crowther et al. 2010).

After inserting eq. 1 into eq. 3 an equation with two unknowns, k and m_{\max} , emerges. Therefore, an additional constraint is necessary in order to solve these equations. Such a constraint is given by the mass of the cluster, M_{ecl} .

$$M_{\text{ecl}} = \int_{m_{\text{low}}}^{m_{\max}} m \cdot \xi(m) dm. \quad (4)$$

The low mass end of the IMF, m_{low} , is set to $0.01 M_{\odot}$. Eqs. 3 and 4 can not be solved explicitly but the existence of a unique solution can be proven (Pflamm-Altenburg & Kroupa 2006).

The left panel of Fig. 1 shows the numerical solution of the system of equations as a solid (with $m_{\text{max}*} = 150 M_{\odot}$) and a dashed line ($m_{\text{max}*} = 300 M_{\odot}$) together with observations of most-massive stars in star clusters (Weidner & Kroupa 2004, 2006; Weidner et al. 2010a). The observations in the left panel of Fig. 1 are also an indication that stars are not randomly sampled from the IMF in a star cluster (Weidner & Kroupa 2006; Haas & Anders 2010). Therefore, physical processes might link the formation of the most-massive star to the potential of the proto-cluster cloud core.

2.3. The relation between the SFR and the most-massive star cluster

In order to calculate the IGIMF for a galaxy it is needed to know how a star cluster population is build-up in a galaxy.

The aim now is to estimate the star-formation rate (SFR) required to build a complete young star-cluster population in one star-formation epoch such that it is populated fully with masses ranging up to $M_{\text{ecl,max}}$. Observational surveys suggest the embedded-cluster mass function (ECMF) is a power-law,

$$\xi_{\text{ecl}}(M_{\text{ecl}}) = k_{\text{ecl}} \cdot \left(\frac{M_{\text{ecl}}}{M_{\text{ecl,max}}} \right)^{-\beta}, \quad (5)$$

with $\beta \approx 2$ (Elmegreen & Efremov 1997; Kroupa 2002; Kroupa & Boily 2002; Lada & Lada 2003). For the total mass of a population of young stellar clusters formed in a time-span δt ,

$$M_{\text{tot}} = \int_{M_{\text{ecl,min}}}^{M_{\text{ecl,max}}} M_{\text{ecl}} \cdot \xi_{\text{ecl}}(M_{\text{ecl}}) dM_{\text{ecl}}, \quad (6)$$

where $M_{\text{ecl,max}}$ is the mass of the heaviest cluster in the population. The normalisation constant k_{ecl} is determined by stating that $M_{\text{ecl,max}}$ is the single most massive cluster,

$$1 = \int_{M_{\text{ecl,max}}}^{\infty} \xi_{\text{ecl}}(M_{\text{ecl}}) dM_{\text{ecl}}. \quad (7)$$

With an ECMF power-law index of $\beta = 2$ we get from eq. 7,

$$k_{\text{ecl}} = \frac{1}{M_{\text{ecl,max}}}. \quad (8)$$

Inserting this into eq. 6 (again with $\beta = 2$),

$$M_{\text{tot}} = M_{\text{ecl,max}} \cdot (\ln M_{\text{ecl,max}} - \ln M_{\text{ecl,min}}). \quad (9)$$

$M_{\text{ecl,min}}$ is the minimal cluster mass which we take to be $5 M_{\odot}$ (a small Taurus-Auriga like sub-group). For arbitrary $\beta \neq 2$ eqs. 8 and 9 change to

$$k_{\text{ecl}} = \frac{\beta - 1}{M_{\text{ecl,max}}} \quad (10)$$

and

$$M_{\text{tot}} = (\beta - 1) \cdot M_{\text{ecl,max}}^{\beta-1} \cdot \left(\frac{M_{\text{ecl,max}}^{2-\beta} - M_{\text{ecl,min}}^{2-\beta}}{2 - \beta} \right). \quad (11)$$

Given a SFR, a fully-populated ECMF with total mass M_{tot} is constructed in the time δt ,

$$M_{\text{tot}} = \text{SFR} \cdot \delta t. \quad (12)$$

Thus, dividing M_{tot} by a formation time, δt , of 10 Myr, and varying $M_{\text{ecl,max}}$ between 10^1 and $10^8 M_{\odot}$, results in a theoretical $M_{\text{ecl,max}}(\text{SFR})$ relation which is shown as solid lines ($\beta = 2$ and $\delta t = 10$ and 1 Myr) and a dashed line ($\beta = 2.4$ and $\delta t = 10$ Myr) in the right panel of Fig. 1 (Weidner et al. 2004). Note that significantly other values of δt lead to a wrong slope and a wrong normalisation of the $M_{\text{ecl,max}} = \text{fn}(\text{SFR})$ relation if $\beta = 2.4$. For example, for $\beta = 2$, a δt of 1 Myr is necessary for the relation to reproduce the data.

2.4. The IGIMF

With the IMF and the two so far derived relations, the $M_{\text{ecl}}-m_{\text{max}}$ -, and the SFR- $M_{\text{ecl,max}}$ -relation, it is now possible to formulate and calculate the IGIMF as follows,

$$\xi_{\text{IGIMF}}(m, t) = \int_{M_{\text{ecl,min}}}^{M_{\text{ecl,max}}(\text{SFR}(t))} \xi(m \leq m_{\text{max}}(M_{\text{ecl}})) \xi_{\text{ecl}}(M_{\text{ecl}}) dM_{\text{ecl}}. \quad (13)$$

Thus $\xi(m \leq m_{\text{max}}) \xi_{\text{ecl}}(M_{\text{ecl}}) dM_{\text{ecl}}$ is the stellar IMF, with m_{max} limited by M_{ecl} , contributed by $\xi_{\text{ecl}} dM_{\text{ecl}}$ clusters with mass near M_{ecl} . While $M_{\text{ecl,max}}$ follows from the SFR- $M_{\text{ecl,max}}$ -relation, $M_{\text{ecl,min}} = 5 M_{\odot}$ is adopted. The resulting IGIMF for a number of SFRs is shown in Fig. 2 (Kroupa & Weidner 2003; Weidner & Kroupa 2005, 2006; Pflamm-Altenburg et al. 2007). Recently, the IGIMF has also been applied to starbursts (Weidner et al. 2010b), and is briefly touched upon below in Fig. 3.

Table 1. IGIMF-ingredients

Ingredient	Reference
basic formulation	Kroupa & Weidner (2003)
$m_{\text{max}}-M_{\text{ecl}}$ -relation	Weidner & Kroupa (2004, 2006); Weidner et al. (2010a)
SFR- $M_{\text{ecl,max}}$ -relation	Weidner et al. (2004)
variation with SFR	Weidner & Kroupa (2005); Pflamm-Altenburg et al. (2007)
application to starbursts	Weidner et al. (2010b)

3. Consequences of the IGIMF

Physically, the IGIMF follows from a few natural assumptions: stars do not form in isolation but in groups and clusters and star-formation is not purely stochastic but processes like stellar feedback regulate star-formation and therefore link their environment (cluster mass) with the final product (most-massive star).

Mathematically, the set of equations developed above from first principles lead to a remarkably successful description of star formation in clusters and galaxies.

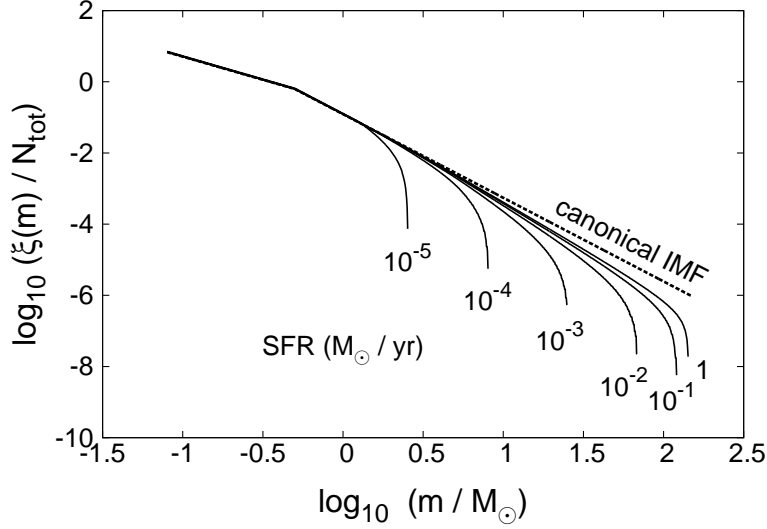


Figure 2. The IGIMF for different SFRs in M_{\odot}/yr . Each IGIMF is normalised such that $\int \xi_{\text{IGIMF}}(m) dm = 1$.

It can be seen in Fig. 2 that the IGIMF is generally steeper than the IMF seen in individual star clusters. This has profound consequences for galaxies. Kroupa & Weidner (2003) showed that the supernova type II rate can be influenced strongly by the IGIMF. This has a significant impact on the chemical evolution of galaxies and has been studied by Köppen et al. (2007) who show that the mass-metallicity relation resulting from the IGIMF agrees well with observational results. Other studies arrive at similar results (Recchi et al. 2009). With the IGIMF the mass-metallicity properties of galaxies thus emerge very naturally.

As is shown in another contribution in these proceedings (Pflamm-Altenburg et al.), the IGIMF predicts a discrepancy between SFRs measured by UV and by $H\alpha$ (Pflamm-Altenburg et al. 2007, 2009) again naturally explaining recent observational claims of such a difference (Meurer et al. 2009; Lee et al. 2009). In addition to these results it is possible to expand the IGIMF theory towards local rather than global properties. This local IGIMF (LIGIMF) explains well the observed radial $H\alpha$ cut-off in disk galaxies (Pflamm-Altenburg & Kroupa 2008).

Furthermore, it can be shown that the IGIMF theory does not necessarily only produce steep (bottom heavy) galaxy-wide IMFs but also top-heavy IGIMFs are possible: When assuming that at very high SFRs extremely massive star clusters are formed it has been shown that these will most likely produce top-heavy IMFs. As star clusters are generally of similar physical size regardless of their mass ($r_{\text{ecl}} \lesssim 1$ pc, Testi et al. 1998; Kroupa 2005; Gutermuth et al. 2005; Rathborne et al. 2006; Scheepmaker et al. 2007), and as proto-stars are much larger than main-sequence stars (with radii between 5000 and 20000 AU, Bacmann et al. 2001; Furuya et al. 2006; van der Tak 2000), crowding of the proto-stars can happen and might change the slope of the high-mass IMF in such clusters (Bonnell et al. 1998; Elmegreen & Shadmehri 2003; Shadmehri 2004; Dabringhausen et al. 2010). If sufficient quantities of such massive star clusters are formed, e.g. in a starburst, the whole IGIMF can become top-heavy (Weidner et al.

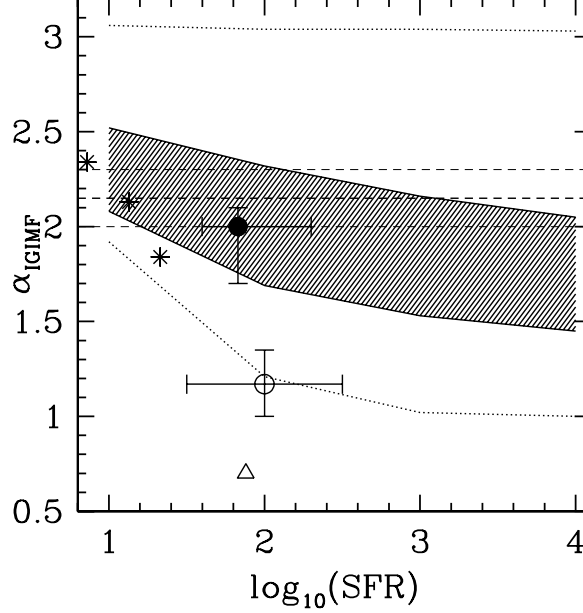


Figure 3. IGIMF slopes above $1 M_{\odot}$ (α_{IGIMF}) when assuming top-heavy starburst clusters and some observational constraints in dependence of the total SFR. The shaded region between the solid lines marks the range of model results for an ECMF slope $\beta = 2.0$ while the dotted lines show the full envelope constrained by all models. The three asterisks are from Davé (2008), the one triangle around $\alpha_3 \approx 0.7$ is from van Dokkum (2008), the filled circle with error bars is from Ballero et al. (2007) while the open circle with error bars corresponds to the Baugh et al. (2005) result. The dashed horizontal line marks the Wilkins et al. (2008a) constraint with the light dashed lines 0.15 dex above and below being their uncertainty range.

2010b). The resulting IGIMFs slopes above $1 M_{\odot}$ for top-heavy IMFs in starbursts are shown in Fig. 3 together with various observational estimates. Indeed, in order to describe the cosmological evolution of stellar mass in galaxies a top-heavy IGIMF seems to be needed (Wilkins et al. 2008a,b).

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